BT-MRE-I & II-05.15-0557 Reg. No. | | | | | | | | |

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B. Tech. Degree I &II Semester Examination in Marine Engineering May 2015

MRE 1101 ENGINEERING MATHEMATICS I

Time: 3 Hours Maximum Marks: 100

(5×20=100)

I. (a) State and prove Rolle's theorem.

(b) State and prove Mean value theorem.

(c) Find the values of a and b such that $\underset{x \to 0}{\ell t} \dfrac{x(1+a\cos x)-b\sin x}{x^3}=1$.

OR

II. (a) Find the radius of curvature of the cycloid $x=a(\theta+\sin\theta)$; $y=a(1-\cos\theta)$.

(b) If $y=\sin(m\sin^{-1}x)$, prove that $(1-x^2)y_2-xy_1+m^2y=0$. Differentiate the above equation n times with respect to x by using Leibnitz's formula.

(c) Find the n^{th} derivative of $\sin x(3x+4)^3$.

III. (a) State Euler's theorem and verify it for the function $u=\dfrac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}$.

(b) If $u=x^y$, show that $\dfrac{\partial^3 u}{\partial x^2\,\partial y}=\dfrac{\partial^3 u}{\partial x\,\partial y\,\partial x}$.

(c) If $r=\sqrt{x^2+y^2}$, $\theta=\tan^{-1}\dfrac{y}{x}$ evaluate $\dfrac{\partial(r,\theta)}{\partial(x,y)}$.

OR

IV. (a) Discuss the maxima and minima of $x^3y^2(1-x-y)$.

(b) If $U=(x^2+y^2+z^2)^{-1/2}$ prove that $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}=0$.

(c) Find the percentage error in the area of a rectangle when on error of +1 percent is made in measuring its length and breadth.

V. (a) Find the equation of the chord joining the points t_1 and t_2 on the parabola $y^2=4ax$. Hence deduce the equation of the tangent at t_1.

(b) Derive the standard equation of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

(c) Find the equation of the hyperbola whose focus is (2, 2), eccentricity is 3 and directrix is $3x-4y=10$.

OR

(P.T.O.)

VI. (a) Find the equation to the parabola whose focus is the point (3, 4) and directrix is the straight line $2x-3y+4=0$.

(b) Find the condition that the straight line $lx+my+n=0$ may touch the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(c) Find the asymptotes of the hyperbola $2x^2+5xy+2y^2+4x+5y=0$.

VII. (a) Find a reduction formula for $\int \sin^m x \cos^n x \, dx$ and hence evaluate $\int_{-\pi}^{\pi} \sin^4 x \cos^2 x \, dx$.

(b) Find the area of the loop of the curve $x^3+y^3=3axy$.

OR

VIII. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2} \, dy \, dx$.

(b) Show that the surface area of the sphere $x^2+y^2+z^2=a^2$ is $4\pi a^2$.

(c) Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} \left(x^2+y^2+z^2\right) dx \, dy \, dz$.

IX. (a) If three vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar then prove that their scalar triple product is zero.

(b) Explain linearly dependent and independent vectors with examples.

(c) Prove that $\left[\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}\right]=\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$.

OR

X. (a) Show that $\nabla \cdot \left(r^n \vec{r}\right)=(n+3)\, r^n$ where $\vec{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$.

(b) If $\vec{A}$ and $\vec{B}$ are irrotational prove that $\vec{A}\times\vec{B}$ is solenoidal.

(c) Prove that $\nabla^2\left(r^n\right)=n(n+1)\, r^{n-2}$.

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